ECON 204C - Macroeconomic Theory

Optimal taxes on fossil fuel in general equilibrium (Golosov et al. 2014)

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Learning Objective

- Optimal taxes on fossil fuel in general equilibrium
  - Decentralized economy
  - Planning problem
- Aggregating firms with the same technology

Decentralized Economy - Consumer

$$\max_{\{C_t, N_t, K_{t+1}\}_{t=0}^{\infty}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t)$$

$$s.t. \quad \mathbb{E}_0 \sum_{t=0}^{\infty} q_t (C_t + K_{t+1}) = \mathbb{E}_0 \sum_{t=0}^{\infty} q_t \Big((1 + r_t - \delta) K_t + w_t N_t + T_t \Big) + \Pi$$

$$(C_t, N_t, K_{t+1}) \in \mathbb{R}^3_+ \qquad \forall t = 0, 1, \cdots$$

- r_t is the net rental rate of capital.
- w_t is the wage rate.
- T_t is a government (lump-sum) transfer.
- Π are the profits from the energy sectors which (in general) are positive because owenership of the scarce resource has value.
- q_t are Arrow-Debreu prices.

Decentralized Economy - Final Goods Producer

$$\Pi_0 \equiv \max_{\{K_{0,t}, N_{0,t}, \mathbf{E}_{0,t}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} q_t \left[F_{0,t}(K_{0,t}, N_{0,t}, \mathbf{E}_{0,t}, S_t) - r_t K_{0,t} - w_t N_{0,t} - \sum_{j=1}^{I} p_{j,t} E_{0,j,t} \right]$$

$$s.t. \qquad (K_{0,t}, N_{0,t}, \mathbf{E}_{0,t}) \in \mathbb{R}_+^{I+2} \qquad \forall t = 0, 1, \cdots$$

- $p_{i,t}$ is the price of fuel of type i.

$$\frac{\partial F_{0,t}}{\partial K_{0,t}} = r_t \qquad \frac{\partial F_{0,t}}{\partial N_{0,t}} = w_t \qquad \frac{\partial F_{0,t}}{\partial E_{0,j,t}} = p_{j,t}$$

(FOC w.r.t. $K_{0,t}$, $N_{0,t}$, $E_{0,j,t}$)

Decentralized Economy - Energy Producer

$$\Pi_i \equiv \max_{\{K_{i,t},N_{i,t},E_{i,t},\mathbf{E}_{i,t},R_{i,t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} q_t \left[(p_{i,t} - \tau_{i,t})E_{i,t} - r_t K_{i,t} - w_t N_{i,t} - \sum_{j=1}^{I} p_{j,t} E_{i,j,t} \right]$$

$$\begin{aligned} s.t. \quad & E_{i,t} = F_{i,t}(K_{i,t}, N_{i,t}, \mathbf{E}_{i,t}, R_{i,t}) \quad \forall t = 0, 1, \cdots \\ & R_{i,t+1} = R_{i,t} - E_{i,t} \quad \forall t = 0, 1, \cdots \\ & (K_{i,t}, N_{i,t}, E_{i,t}, \mathbf{E}_{i,t}, R_{i,t+1}) \in \mathbb{R}_{+}^{I+4} \quad \forall t = 0, 1, \cdots \end{aligned}$$

- $\tau_{i,t}$ is a per-unit tax on the resource.
- Total profits are $\Pi = \sum_{i=0}^{I} \Pi_i$.

Profit Maximization for Energy Producer

$$\Pi_i \equiv \max_{\{K_{i,t},N_{i,t},E_{i,t},\mathbf{E}_{i,t},R_{i,t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} q_t \left[(p_{i,t} - \tau_{i,t})E_{i,t} - r_t K_{i,t} - w_t N_{i,t} - \sum_{j=1}^{I} p_{j,t} E_{i,j,t} \right]$$
$$+ \sum_{t=0}^{\infty} q_t \hat{\lambda}_{i,t} \Big[F_{i,t}(K_{i,t}, N_{i,t}, \mathbf{E}_{i,t}, R_{i,t}) - E_{i,t} \Big]$$
$$+ \sum_{t=0}^{\infty} q_t \hat{\mu}_{i,t} \Big[R_{i,t} - E_{i,t} - R_{i,t+1} \Big]$$
$$+ \sum_{t=0}^{\infty} q_t \Big[\cdots - \hat{\xi}_{i,t} E_{i,t} + \cdots \Big]$$

$$\hat{\lambda}_{i,t} \frac{\partial F_{i,t}}{\partial K_{i,t}} = r_t \qquad \hat{\lambda}_{i,t} \frac{\partial F_{i,t}}{\partial N_{i,t}} = w_t \qquad \hat{\lambda}_{i,t} \frac{\partial F_{i,t}}{\partial E_{i,j,t}} = p_{j,t} \qquad \text{(FOC w.r.t. } K_{i,t},\ N_{i,t},\ E_{i,j,t}\text{)}$$

$$\hat{\lambda}_{i,t} + \hat{\mu}_{i,t} + \hat{\xi}_{i,t} = (p_{i,t} - \tau_{i,t}) \qquad \text{(FOC w.r.t. } E_{i,t}\text{)}$$

We assume that the tax proceeds are rebated lump-sum to the representative consumer.

$$T_t = \sum_{i=1}^{I} \tau_{i,t} E_{i,t}$$

$\tilde{S}_t$ maps a history of anthropogenic emissions into the current level of atmospheric carbon concentration, S_t. The history is defined to start at the time of industrialization, a date defined as $-T$:

$$S_t = \tilde{S}_t \left(\sum_{i=1}^{i_g - 1} E_{i,-T}, E^f_{-T+1}, \cdots, E^f_t \right) \quad \text{where} \quad E^f_s \equiv \sum_{i=1}^{i_g - 1} E_{i,s}$$

Competitive Equilibrium

Definition A competitive equilibrium consists of an allocation $\{C_t, N_t, K_{t+1}, K_{0,t}, N_{0,t}, \mathbf{E}_{0,t}, \{K_{i,t}, N_{i,t}, E_{i,t}, \mathbf{E}_{i,t}, R_{i,t+1}\}_{i=1}^{I}, S_t\}_{t=0}^{\infty}$, a set of prices $\{q_t, r_t, w_t, \mathbf{p}_t\}$, and a set of policies $\{\boldsymbol{\tau}_t, T_t\}$ such that:

1. the allocations solve the consumer's and the firms' problems given prices and policies,
2. the government budget constraint is satisfied in every period,
3. the current level of atmospheric carbon concentration S_t satisfies the carbon cycle constraint in every period, and
4. markets clear.

Planning Problem

$$
\begin{aligned}
&\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t) \\
s.t. \quad & C_t + K_{t+1} = F_{0,t}(K_{0,t}, N_{0,t}, \mathbf{E}_{0,t}, S_t) + (1-\delta)K_t && \forall t = 0, 1, \cdots \\
& E_{i,t} = F_{i,t}(K_{i,t}, N_{i,t}, \mathbf{E}_{i,t}, R_{i,t}) && \forall i = 1, \cdots, I \text{ and } \forall t = 0, 1, \cdots \\
& R_{i,t+1} = R_{i,t} - E_{i,t} && \forall i = 1, \cdots, I \text{ and } \forall t = 0, 1, \cdots \\
& E_{i,t} = \sum_{j=0}^{I} E_{j,i,t} && \forall i = 1, \cdots, I \text{ and } \forall t = 0, 1, \cdots \\
& K_t = \sum_{i=0}^{I} K_{i,t} && \forall t = 0, 1, \cdots \\
& N_t = \sum_{i=0}^{I} N_{i,t} && \forall t = 0, 1, \cdots \\
& S_t = \tilde{S}_t \left(\sum_{i=1}^{i_g - 1} E_{i,-T}, E^f_{-T+1}, \cdots, E^f_t \right) && \forall t = 0, 1, \cdots \\
& (Nonnegative\ Constraints)
\end{aligned}
$$

Planning Problem

$$
\begin{aligned}
\mathcal{L} = \max \mathbb{E}_0 & \sum_{t=0}^{\infty} \beta^t U(C_t) \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_{0t} \Big[F_{0,t}(K_{0,t}, N_{0,t}, \mathbf{E}_{0,t}, \tilde{S}_t(\cdot)) + (1-\delta) K_t - C_t - K_{t+1} \Big] \\
& + \sum_{i=1}^{I} \Bigg\{ \quad \sum_{t=0}^{\infty} \beta^t \lambda_{it} \Big[F_{i,t}(K_{i,t}, N_{i,t}, \mathbf{E}_{i,t}, R_{i,t}) - E_{i,t} \Big] \\
& \qquad + \sum_{t=0}^{\infty} \beta^t \mu_{it} \Big[R_{i,t} - E_{i,t} - R_{i,t+1} \Big] \\
& \qquad + \sum_{t=0}^{\infty} \beta^t \chi_{it} \Big[E_{i,t} - \sum_{j=0}^{I} E_{j,i,t} \Big] \qquad \Bigg\} \\
& + \sum_{t=0}^{\infty} \beta^t \kappa_t \Big[K_t - \sum_{i=0}^{I} K_{i,t} \Big] \\
& + \sum_{t=0}^{\infty} \beta^t \nu_t \Big[N_t - \sum_{i=0}^{I} N_{i,t} \Big] \\
& + \sum_{t=0}^{\infty} \beta^t \Big[\cdots - \xi_{i,t} E_{i,t} + \cdots \Big]
\end{aligned}
$$

Planning Problem

FOC w.r.t. E_{it} is given by

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \lambda_{0,t+j} \frac{\partial F_{0t+j}}{\partial S_{t+j}} \frac{\partial \tilde{S}_{t+j}}{\partial E_{it}} - \lambda_{it} - \mu_{it} + \chi_{it} - \xi_{it} = 0$$

FOC w.r.t. $E_{0,i,t}$ is given by

$$\lambda_{0t} \frac{\partial F_{0,t}}{\partial E_{0it}} - \chi_{it} = 0$$

FOC w.r.t. $R_{i,t+1}$ is given by

$$-\mu_{it} + \beta \mathbb{E}_t \left[\lambda_{it+1} \frac{\partial F_{i,t+1}}{\partial R_{i,t+1}} + \mu_{it+1} \right] = 0 \qquad \text{(The Hotelling's Rule)}$$

- Lagrange multiplier of a constraint expresses the quantity of utils that could be obtained when relaxing the constraint by one unit. This expressions only holds when choice variables are evaluated at the optima.

Planning Problem

$$\frac{\partial F_{0,t}}{\partial E_{0it}} - \underbrace{\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{0,t+j}}{\lambda_{0t}} (-1) \frac{\partial F_{0t+j}}{\partial S_{t+j}} \frac{\partial \tilde{S}_{t+j}}{\partial E_{it}}}_{=\Lambda^S_{i,t}} = \frac{\lambda_{it} + \mu_{it} + \xi_{it}}{\lambda_{0t}} \qquad \text{(eq. 8 in GHKT)}$$

Each period, planner allocates factors such that for any $i \in \{1, \cdots, I\}$

$$\lambda_{0t} \frac{\partial F_{0,t}}{\partial N_{0,t}} = \lambda_{it} \frac{\partial F_{i,t}}{\partial N_{i,t}} = \nu_t \qquad \Leftrightarrow \qquad \frac{\lambda_{it}}{\lambda_{0t}} = \frac{\frac{\partial F_{0,t}}{\partial N_{0,t}}}{\frac{\partial F_{i,t}}{\partial N_{i,t}}}$$

Then we have

$$\underbrace{\frac{\partial F_{0,t}}{\partial E_{0it}}}_{\text{Private MB}} = \underbrace{\frac{\frac{\partial F_{0,t}}{\partial N_{0,t}}}{\frac{\partial F_{i,t}}{\partial N_{i,t}}}}_{\text{Private MC from labor reallocation}} + \underbrace{\frac{\mu_{it}}{\lambda_{0t}}}_{\text{Private MC from resource extraction}} + \underbrace{\Lambda^S_{i,t}}_{\text{Social MC from carbon emissions}}$$

Implementation

$$\frac{\partial F_{0,t}}{\partial E_{0it}} - \Lambda^S_{i,t} = \frac{\frac{\partial F_{0,t}}{\partial N_{0,t}}}{\frac{\partial F_{i,t}}{\partial N_{i,t}}} + \frac{\mu_{it}}{\lambda_{0t}}$$

$$p_{i,t} - \tau_{i,t} = \hat{\lambda}_{i,t} + \hat{\mu}_{i,t}$$

$$\tau_{i,t} = \Lambda^S_{i,t} \quad \text{(Pigouvian carbon tax)}$$

$$p_{i,t} = \frac{\partial F_{0,t}}{\partial E_{0it}} \quad \text{(FOC w.r.t. } E_{0it} \text{ from final goods sector)}$$

$$\hat{\lambda}_{i,t} = \frac{w_t}{\frac{\partial F_{i,t}}{\partial N_{i,t}}} \quad \text{(FOC w.r.t. } N_{i,t} \text{ from } i^{th} \text{ energy sector)}$$

$$w_t = \frac{\partial F_{0,t}}{\partial N_{0t}} \quad \text{(FOC w.r.t. } N_{0t} \text{ from final goods sector)}$$

$$\hat{\mu}_{i,t} = \frac{\mu_{it}}{\lambda_{0t}}$$

Uniform Tax on Carbon Energy Inputs

- Units of $E_{i,t}$'s are normalized to be in tons of carbon-equivalent, the following holds by construction:

$$\frac{\partial \tilde{S}_{t+j}}{\partial E_{it}} = \frac{\partial \tilde{S}_{t+j}}{\partial E_{jt}} \qquad \forall\ i, j \in \{1, \cdots, I_g - 1\}$$

- Marginal externality damage is independent of energy sector:

$$\Lambda^S_{i,t} = \begin{cases} \underbrace{\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \frac{U'(C_{t+j})}{U'(C_t)} (-1) \frac{\partial F_{0t+j}}{\partial S_{t+j}} \frac{\partial \tilde{S}_{t+j}}{\partial E_{kt}}}_{=\Lambda^S_t} & i \in \{1, \cdots, i_g - 1\} \\ 0 & i \in \{i_g, \cdots, I\} \end{cases}$$

Uniform Tax on Carbon Energy Inputs

$$\begin{aligned}
\Lambda_t^s &= \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \frac{U'(C_{t+j})}{U'(C_t)} (-1) \frac{\partial F_{0t+j}}{\partial S_{t+j}} \frac{\partial \tilde{S}_{t+j}}{\partial E_{it}} \\
&= \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \frac{C_{t+j}^{-1}}{C_t^{-1}} (-1)(-\gamma_{t+j}) Y_{t+j} (1 - d_j) \\
&= Y_t \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \frac{\frac{C_t}{Y_t}}{\frac{C_{t+j}}{Y_{t+j}}} \gamma_{t+j} (1 - d_j) \\
\frac{\Lambda_t^s}{Y_t} &= \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \frac{(1 - s_t)}{(1 - s_{t+j})} \gamma_{t+j} (1 - d_j) \\
&= \overline{\gamma}_t \sum_{j=0}^{\infty} \beta^j \left[\phi_L + (1 - \phi_L) \phi_0 (1 - \phi)^j \right] && (s_t = s_{t+j} \text{ and } \mathbb{E}_t[\gamma_{t+j}] = \overline{\gamma}_t) \\
&= \overline{\gamma}_t \left[\frac{\phi_L}{1 - \beta} + \frac{(1 - \phi_L) \phi_0}{1 - (1 - \phi) \beta} \right] && \text{(eq. 12 in GHKT)}
\end{aligned}$$

Aggregating firms with the same technology

Consider an economy with M firms, indexed by $i = 1, 2, \cdots, M$ which produce a homogeneous good with the same technology $zF(k^i, n^i)$ where z is aggregate productivity. Assume that F is strictly increasing, strictly concave, differentiable in both arguments and constant returns to scale. Can we aggregate these individual firms into a representative firms? Suppose input markets are competitive.

$$\max_{\{k^i, n^i\}} \; zF(k^i, n^i) - wn^i - (r + \delta)k^i$$

$$zF_k(k^i, n^i) = r + \delta$$
$$zF_n(k^i, n^i) = w$$

$$\frac{F_k(k^i, n^i)}{F_n(k^i, n^i)} = \frac{f_k(k^i/n^i)}{f_n(k^i/n^i)} \quad \text{where} \quad f_k(k^i/n^i) = F_k(k^i/n^i, 1) \quad \text{and} \quad f_n(k^i/n^i) = F_n(1, n^i/k^i)$$

Aggregating firms with the same technology

$$\frac{f_k(k^i/n^i)}{f_n(k^i/n^i)} = \frac{r+\delta}{w}$$

$$\frac{k^i}{n^i} = g\left(\frac{r+\delta}{w}\right) \qquad (f_k/f_n \text{ is strictly decreasing in } k^i/n^i)$$

$$\frac{k^i}{n^i} = \frac{K}{N} \qquad \forall\ i = 1, \cdots, M \qquad \left(K = \textstyle\sum_i k^i \text{ and } N = \sum_i n^i\right)$$

$$\begin{aligned} z\sum_{i=1}^{M} F(k^i, n^i) &= z\sum_{i=1}^{M}\left[F_k(k^i, n^i)k^i + F_n(k^i, n^i)n^i\right] && \text{(HD1)} \\ &= z\sum_{i=1}^{M}\left[f_k(k^i/n^i)k^i + f_n(k^i/n^i)n^i\right] \\ &= z\sum_{i=1}^{M}\left[f_k(K/N)k^i + f_n(K/N)n^i\right] \\ &= z\left[f_k(K/N)K + f_n(K/N)N\right] \\ &= z\left[F_k(K, N)K + F_n(K, N)N\right] \\ &= zF(K, N) && \text{(HD1)} \end{aligned}$$

Reference

Golosov, M., Hassler, J., Krusell, P., & Tsyvinski, A. (2014). Optimal taxes on fossil fuel in general equilibrium. Econometrica, 82(1), 41-88.