ECON 204C - Macroeconomic Theory

The Hotelling Model

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Learning Objective

- The Hotelling Rule
  - ⋆ Partial equilibrium (Karp, 2017)
  - ⋆ General equilibrium (Hassler et al., 2019)

- A price-taking firm has discount factor $\rho \in (0,1)$, and faces prices $\{p_t\}_{t=0}^{T}$.

$$\rho = \frac{1}{1+r}$$

- A firm has a fixed stock of the resource, $R_0 > 0$, and must pay stock-dependent extraction cost $c : \mathcal{R} \times \mathcal{E} \to \mathbb{R}$.

$$\frac{\partial c(R,E)}{\partial R} \leq 0, \qquad \frac{\partial c(R,E)}{\partial E} \geq 0, \qquad \frac{\partial^2 c(R,E)}{\partial E^2} \geq 0$$

  - ⋆ A higher stock lowers costs.
  - ⋆ A higher extraction increases costs.
  - ⋆ A higher extraction increases marginal costs.

Optimal Extraction of Nonrenewable Resources

$$\max_{\{E_t\}_{t=0}^{T}} \sum_{t=0}^{T} \rho^t \Big[p_t E_t - c(R_t, E_t) \Big]$$

$$\begin{aligned} s.t. \quad & E_t \in [0, R_t] && \forall\ t = 0, \cdots, T \\ & R_{t+1} = R_t - E_t && \forall\ t = 0, \cdots, T \\ & R_0 > 0 \text{ given} \end{aligned}$$

Hotelling Rule (Euler Equation)

For any two adjacent periods where extraction is positive, we have

$$p_t - \frac{\partial c(R_t, E_t)}{\partial E_t} = \rho \Big[\underbrace{p_{t+1} - \frac{\partial c(R_{t+1}, E_{t+1})}{\partial E_{t+1}}}_{(A)} \underbrace{- \frac{\partial c(R_{t+1}, E_{t+1})}{\partial R_{t+1}}}_{(B)} \Big]$$

- LHS is the gains from additional extraction in period t.
- RHS is the present value of the losses from additional extraction in period t.
  1. Term (A) is the reduction in period $t+1$ profit, the loss arising from having one less unit to extract in period $t+1$.
  2. Term (B) is the increase in period $t+1$ extraction cost due to a reduction in the stock at the beginning of period $t+1$.

Constant Marginal Costs

Assume that extraction cost is stock-independent and constant with respect to extraction: $c(R, E) = cE$. Then

$$\underbrace{p_t - c}_{Rent_t} = \rho(\underbrace{p_{t+1} - c}_{Rent_{t+1}})$$

- The present value of rent is the same in any two periods where extraction is positive.

$$(p_t - c) = \rho^j (p_{t+j} - c) \quad \Leftrightarrow \quad (1+r)^j (p_t - c) = (p_{t+j} - c) \qquad \text{(No Intertemporal Arbitrage)}$$

  Firms cannot increase their payoff by moving extraction between any two periods where extraction is positive.

- The value of nonrenewable resources equals the initial rent times the initial stock.

$$V(R_0) = \sum_{t=0}^{T} \underbrace{\rho^t (p_t - c)}_{p_0 - c} E_t^* = (p_0 - c) \sum_{t=0}^{T} E_t^* = (p_0 - c) R_0$$

Constant Marginal Costs

$$(p_t - c) = \frac{1}{1+r}(p_{t+1} - c) \quad \Leftrightarrow \quad (1+r)(p_t - c) = (p_{t+1} - c) \quad \Leftrightarrow \quad (p_{t+1} - c) - (p_t - c) = r(p_t - c)$$

- Rent rises at the rate of interest.

$$\frac{(p_{t+1} - c) - (p_t - c)}{(p_t - c)} = r \quad \Leftrightarrow \quad \frac{p_{t+1} - p_t}{p_t} = r - \frac{rc}{p_t}$$

1. Price rises at less than the rate of interest if $c > 0$.
2. Price rises at the rate of interest if $c = 0$.

$$\frac{p_{t+1}}{p_t} = 1 + r$$

The Hotelling Rule under General Equilibrium (Hassler et al., 2019)

- Assume that capital depreciates fully between periods and that the production function is of the Cobb-Douglas form

$$Y_t = F(K_t, E_t, L_t) = K_t^{\alpha} E_t^{\nu} L_t^{1-\nu-\alpha}$$

  Since labor is not going to be important for the analysis, we normalize it to unity.
- Assume that the utility function is logarithmic in consumption

$$U(C_0, C_1) = \ln C_0 + \beta \ln C_1$$

- With $\bar{K}_0$ given, the problem is to choose the variables to maximize the discounted utility in the two periods subject to the resource constraints.

$$E_0 + E_1 \leq \bar{R}_0 \qquad \text{(Nonrenewable Resource)}$$

$$C_0 + K_1 \leq \bar{K}_0^{\alpha} E_0^{\nu} \qquad \text{(Income in 0)}$$

$$C_1 \leq K_1^{\alpha} E_1^{\nu} \qquad \text{(Income in 1)}$$

  Total extraction equals (does not exceed) the initial stock. Consumption plus savings equals (does not exceed) income.

Welfare Maximization

$$\max_{K_1, E_0} \ \ln\left(\bar{K}_0^{\alpha} E_0^{\nu} - K_1\right) + \beta \ln\left(K_1^{\alpha} (\bar{R}_0 - E_0)^{\nu}\right) \qquad \text{(Welfare)}$$

$$\frac{1}{C_0} = \beta \frac{1}{C_1} \alpha \frac{Y_1}{K_1} \qquad \text{(FOC w.r.t. } K_1\text{)}$$

$$\frac{1}{C_0} \nu \frac{Y_0}{E_0} = \beta \frac{1}{C_1} \nu \frac{Y_1}{E_1} \qquad \text{(FOC w.r.t. } E_0\text{)}$$

Rewriting the two equations above, we get

$$\frac{C_1}{\beta C_0} = \alpha \frac{Y_1}{K_1} \quad \text{and} \quad \frac{C_1}{\beta C_0} = \frac{\nu \frac{Y_0}{E_0}}{\nu \frac{Y_1}{E_1}} \quad \Rightarrow \quad \frac{\nu \frac{Y_0}{E_0}}{\nu \frac{Y_1}{E_1}} = \alpha \frac{Y_1}{K_1} \qquad \text{(Hotelling Rule)}$$

Competitive Input Markets

$$\max_{K_t, E_t, L_t} K_t^{\alpha} E_t^{\nu} L_t^{1-\nu-\alpha} - (\delta + r_t)K_t - p_t E_t - w_t L_t \quad \text{where} \quad \delta = 1$$

$$1 + r_t = \alpha \frac{Y_t}{K_t}, \quad p_t = \nu \frac{Y_t}{E_t}, \quad w_t = (1 - \nu - \alpha)\frac{Y_t}{L_t} \tag{FOC}$$

$$\frac{\nu \frac{Y_0}{E_0}}{\nu \frac{Y_1}{E_1}} = \alpha \frac{Y_1}{K_1} \quad \Leftrightarrow \quad \frac{p_1}{p_0} = 1 + r_1 \tag{Hotelling Rule}$$

- There are two ways of transferring consumption opportunities between periods.
  1. The first is to save in the form of capital.
  2. The second is to save in the form of nonrenewable resource—not using a unit of nonrenewable resource today and instead using it next period.
- It cannot be optimal to use two ways of saving in a way that one gives more returns than the other—they must thus in an optimal allocation have the same return.

Implication

- The Hotelling formula is key to understanding the economics of resources that have the dynamic property that using a unit today reduces available resources in the future.

- According to the theorem, the oil price should grow at the rate of the interest rate. The Hotelling rent is thus the maximum rent that could be obtained while depleting the nonrenewable resource.

Reference

Karp, L. (2017). Natural Resources as Capital: Theory and Policy. MIT press.

Hassler, J., Krusell, P., & Olovsson, C. (2019). The Climate and the Economy.